

Third family hypercharge model for $R_{K^{(*)}}$ and aspects of the fermion mass problem

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ABSTRACT: We present a model to explain LHCb's recent measurements of R_K and R_{K^*} based on an anomaly-free, spontaneously-broken $U(1)_F$ gauge symmetry, without any fermionic fields beyond those of the Standard Model (SM). The model explains the hierarchical heaviness of the third family and the smallness of quark mixing. The $U(1)_F$ charges of the third family of SM fields and the Higgs doublet are set equal to their respective hypercharges. A heavy Z' particle with flavour-dependent couplings can modify the $[\bar{b}_L \gamma^\rho s_L][\bar{\mu}_L \gamma_\rho \mu_L]$ effective vertex in the desired way. The Z' contribution to $B_s - \bar{B}_s$ mixing is suppressed by a small mixing angle connected to V_{ts} , making the constraint coming from its measurement easier to satisfy. The model can explain R_K and R_{K^*} whilst simultaneously passing other constraints, including measurements of the lepton flavour universality of Z couplings.

KEYWORDS: Beyond Standard Model, Quark Masses and SM Parameters, Heavy Quark Physics

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1 Introduction

Recent measurements of semi-leptonic B -meson decays by the LHCb collaboration in LHC Run I [1, 2] suggest that they violate electron-muon universality more than is predicted by the SM [3]. The primary evidence comes from $B \rightarrow K^{(*)}l^+l^-$ where $l \in \{e, \mu\}$, as encapsulated by the R_K and R_{K^*} parameters:

$$R_K = \frac{BR(B \rightarrow K\mu^+\mu^-)}{BR(B \rightarrow Ke^+e^-)}, \quad R_{K^*} = \frac{BR(B \rightarrow K^*\mu^+\mu^-)}{BR(B \rightarrow K^*e^+e^-)}. \quad (1.1)$$

There are three such discrepant measurements: R_K in a di-lepton invariant mass squared bin of $Q^2 \in [1, 6] \text{ GeV}^2$ which disagrees with the SM at 2.6σ , $R_{K^*}(Q^2 \in [0.045, 1.1] \text{ GeV}^2)$ which has a 2.2σ level discrepancy with the SM prediction and $R_{K^*}(Q^2 \in [1.1, 6] \text{ GeV}^2)$, which is at odds with the SM prediction at the 2.5σ level. Each individual measurement is not especially significant, but their combination is around the 4σ level. The prediction of R_K and R_{K^*} in the SM is particularly clean, since the theoretical uncertainties cancel nicely in the ratios to leave only a very small overall uncertainty. Analyses of Run II data are eagerly awaited, as are similar measurements from BELLE II, but in the meantime many have tried to explain the discrepancies with a new physics effect. If we quantify the

effect of new physics by a change to the Wilson coefficient of a single effective field theory (EFT) operator,

$$\mathcal{O} \propto [\bar{s}\gamma_\mu P_L b] [\bar{\mu}\gamma^\mu P_X \mu] \quad (1.2)$$

is found to fit R_K , R_{K^*} and other B -physics data [4], where $P_X = 1$ (i.e. a vector-like coupling to muon pairs) or $P_X = P_L$ (a left-handed coupling to muon pairs). $P_X = P_R$ does *not* provide a good fit. This EFT operator may arise from integrating out some heavy new particle which preferentially couples to muons rather than electrons. The fits indicate that the mass of the new particle is 31 TeV divided by the square root of a product of two couplings. Since the couplings are flavour dependent, this raises the exciting possibility of experimentally probing new physics *which could potentially explain the pattern of hierarchies in fermion masses and mixings*. In the spirit of simplified model building, one begins by looking for a single new particle to explain the data. It is found at tree-level that this new particle [5]¹ could either be a flavour-dependent leptoquark or a Z' with flavour dependent couplings [11–38]. It is the latter possibility that we focus on in the present paper.

Here, our *modus operandi* is to incrementally model-build the Z' simplified model toward a more fundamental theory. One obvious choice is to take the Z' to be the heavy gauge boson from an underlying $U(1)_F$ flavoured gauge symmetry.² For example, in the Third Family Hypercharge Model, we will arrange the $U(1)_F$ charges of the SM fermions such that only the third family is allowed a gauge-invariant Yukawa coupling at the renormalisable level.³ The first two families and neutrinos may then acquire Yukawa couplings (or masses) at the non-renormalisable level. Fermion mixing may also be generated by such non-renormalisable terms, for example by the Froggatt-Nielsen mechanism [40]. To be guaranteed a consistent quantum field theory, one chooses the charges such that no anomalies (including mixed anomalies and gauge-gravity anomalies) arise. This is a highly non-trivial constraint on possible $U(1)_F$ charges.

Our path then is clear: we wish to find anomaly-free combinations of $U(1)_F$ charges which predict that the terms in eq. (1.2) are present, as well as the third family Yukawa couplings (but no other Yukawa couplings at the renormalisable level). We find that there is indeed such an anomaly-free set of $U(1)_F$ charges which satisfies these criteria, without the need to introduce any additional fermion fields beyond the SM content. Moreover, this solution to the anomaly constraints is unique up to normalisation, with the $U(1)_F$ charge of each third family field proportional to its hypercharge.⁴ After this, we wish to make

¹Other approaches based on more complete model set-ups have been discussed, for example composite Higgs [6, 7], composite leptoquark [8], or warped extra dimensional [9, 10] models.

²By the word “flavoured”, we here mean that the gauge field for the $U(1)_F$ symmetry has flavour-dependent couplings to the SM fermions.

³Third family hypercharge times another $U(1)$ gauge symmetry (first two family hypercharge) were simultaneously broken to the diagonal subgroup $U(1)_Y$ in ref. [39]. The model was not connected to any B -anomalies (it was invented before the relevant measurements) but it does address some aspects of the fermion mass problem after adding either additional Higgs doublets or an additional vector-like family to the SM.

⁴The space of solutions to the anomaly constraints has been recently studied in ref. [41], subject to the constraint that the right-handed down-type quarks all have vanishing $U(1)_F$ charge. With such a constraint,

sure that the model passes all existing constraints upon it, since it will necessarily predict additional Z' couplings to other fermions than just those in eq. (1.2): for example, it should also be possible in the model to somehow reduce the Z' coupling to $\bar{s}\gamma_\mu P_R b$, which fits the B -physics data badly if it dominates over $\bar{s}\gamma_\mu P_L b$.

Somewhat similar approaches (which also aim to connect the recent B -physics data with the hierarchies in the fermion masses or their mixing angles) have been made in the literature. As we shall now discuss, our approach is significantly different to these, both in its aims and in its construction. In refs. [42, 43] different anomaly-free sets of gauged $U(1)_F$ charges were found in models with additional fermionic SM singlet fields and an additional Higgs doublet. These models allow enough Yukawa couplings at the renormalisable level to achieve quark and lepton mixing, as well as yielding the effective field theory operator in eq. (1.2). Refs. [30, 31] are similar in spirit to refs. [42, 43], except that mixing between the third family and the first two is banned at the renormalisable level in the former two papers. However, the models shed no light on the origin of the hierarchy in fermion masses. Thus, refs. [30, 31, 42, 43] are quite different to our approach in which, by requiring that only the third family fermions are allowed Yukawa couplings at the renormalisable level, we provide a possible explanation for the hierarchical heaviness of the third family and of the small size of quark mixing.

In ref. [24], a spontaneously broken $U(2)_F \cong SU(2)_F \times U(1)_F$ flavour symmetry, in which the $U(1)_F$ subgroup is gauged, was used to explain $R_{K^{(*)}}$ via the flavour-dependent interactions of the corresponding Z' . By introducing scalar spurions that parametrize the $U(2)_F$ breaking, the authors of ref. [24] are able to arrange appropriate power-law hierarchies for the fermion masses and mixing angles, from $\mathcal{O}(1)$ renormalisable fundamental couplings à la Froggatt-Nielsen [40]. The model fits the $R_{K^{(*)}}$ measurements by increasing the denominators in eq. (1.1) whilst simultaneously decreasing the numerators. Increasing the denominators is (by now) somewhat disfavoured by global fits to various B data. Furthermore, as is typical in Froggatt-Nielsen inspired model-building, the light generation quarks carry the largest charges under the flavoured $U(1)_F$ symmetry. Consequently, the Z' boson in such a model couples most strongly to the valence quarks u , d , and s , and is therefore subject to stronger constraints from current data (for example from the high- p_T dilepton tails [44] in pp collisions). Nonetheless, the model of ref. [24] remains similar in its aims to ours and, indeed, goes further into detail on the fermion mass model-building by explicitly writing down the higher dimension operators responsible for the light fermion masses (and mixings), at the level of an effective description involving SM fields and spurions.

The model proposed in ref. [45] also seeks to connect the $R_{K^{(*)}}$ measurements with the fermion mass hierarchies through a gauged and spontaneously broken $U(1)_F$ symmetry with flavour-dependent couplings. In that model, as well as an additional Higgs doublet, a vector-like fourth family of SM fermions was introduced to produce the required operator eq. (1.2). The vector-like fourth family is the only one charged under $U(1)_F$, meaning

it is shown in ref. [41] that one must introduce additional fermions (which may be identified as dark matter candidates) to satisfy the anomaly equations. In the present work, we evade this conclusion by allowing b_R to have a $U(1)_F$ charge, and so we are able to find an anomaly-free set of charges with only SM fields.

$F_{Q'_i} = 0$	$F_{u_{R'_i}} = 0$	$F_{d_{R'_i}} = 0$	$F_{L'_i} = 0$	$F_{e_{R'_i}} = 0$	$F_H = -1/2$
$F_{Q'_3} = 1/6$	$F_{u'_{R3}} = 2/3$	$F_{d'_{R3}} = -1/3$	$F_{L'_3} = -1/2$	$F_{e'_{R3}} = -1$	

Table 1. $U(1)_F$ charges of the fields in the Third Family Hypercharge Model, where $i \in \{1, 2\}$. All gauge anomalies, mixed gauge anomalies and mixed gauge-gravity anomalies cancel.

that gauge anomalies are cancelled. Whilst the existence of a gauged $U(1)_F$ symmetry and a connection between fermion mass predictions and the B -discrepancies is in common with refs. [45], our model differs in that it is anomaly-free without adding any matter fields or Higgs doublet fields to the SM field content. In that sense, our model is a more minimal extension to the SM. An attempt has also been made to connect the $R_{K^{(*)}}$ measurements with the fermion mass hierarchies through a leptoquark model, rather than a Z' , in ref. [46].⁵

We note that another paper introduced a simplified⁶ Z' model where the Z' coupled dominantly to left-handed bottom quarks and to left-handed muons [47]. No attempt was made to solve any anomaly constraints or to explain aspects of the observed fermion masses and mixings, and so we construct a more complete model here.

2 Third family hypercharge model

In order to ban all Yukawa couplings except those of the third family, we set the $U(1)_F$ charges of the first two families to zero but give the Higgs H a non-zero charge. With this constraint, the only set of charges that satisfies all of the anomaly equations is the one where fermion charges of the third family are proportional to their hypercharges. Since it is well known that hypercharges fit into grand unified groups such as $SU(5)$ and $SO(10)$, the $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_F$ gauge symmetry may be embedded within some larger non-abelian unified symmetry. We list the charges in table 1. At the renormalisable level, the only allowed Yukawa couplings are

$$\mathcal{L} = Y_t \overline{Q_{3L}} H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + Y_\tau \overline{L_{3L}} H^c \tau'_R + \text{H.c.}, \quad (2.1)$$

thus explaining the relative hierarchical heaviness of the third family⁷ once the neutral component of the Higgs doublet acquires its vacuum expectation value v . Under the SM

⁵In this leptoquark model, a *global* (*i.e.* not gauged) $U(1)_F$ symmetry is invoked. The assignment of $U(1)_F$ charges to the SM fermions is arranged so as to produce mass hierarchies via the Froggatt-Nielsen mechanism, while the assignment of $U(1)_F$ charges to the leptoquarks leads to a hierarchy in the leptoquark couplings which is capable of explaining the B -physics data. Note that, in this model, the $U(1)_F$ charges of the leptoquarks can be chosen independently to the SM fermion charges, such that the explanations for the B -physics data and for the fermion masses effectively decouple into two independent explanations. In contrast, in our model, we only have the SM fermions, whose assignment of $U(1)_F$ charges can provide a *shared* explanation for both the B -physics data and the fermion masses, which is moreover anomaly-free.

⁶There have been ultra-simplified Z' models in the literature, e.g. [11, 23], where only couplings to muon flavoured leptons and $\bar{s}b + \text{H.c.}$ have been considered. These do not preserve $SU(2)_L$.

⁷ Y_t , Y_b and Y_τ are complex dimensionless Yukawa couplings.

gauge symmetry $SU(3) \times SU(2)_L \times U(1)_Y$, the fields transform as $H \sim (1, 2, -1/2)$,

$$Q'_{iL} \sim (3, 2, 1/6), \quad L'_{iL} \sim (1, 2, -1/2), \quad u'_{iR} \sim (3, 1, 2/3), \quad d'_{iR} \sim (3, 1, -1/3), \quad e'_{iR} \sim (1, 1, -1),$$

where we suppress gauge indices but not the family index $i \in \{1, 2, 3\}$ and $H^c = (H^+, -H^{0*})^T$. Weak eigenbasis fermion fields are written with a prime, whereas fermion fields in the mass eigenbasis shall be written without a prime. The only Yukawa terms allowed are precisely those in eq. (2.1). More detailed model building may provide estimates for neutrino and lighter family masses, and fermion mixings, which may come from non-renormalisable operators. A small perturbation of eq. (2.1) from such non-renormalisable operators will necessarily predict small quark mixing. For now, we shall simply constrain fermion mixings and the masses of the first two generations to be at their central measured values.

2.1 Masses of gauge bosons and Z - Z' mixing

The $U(1)_F$ symmetry is assumed to be spontaneously broken by a SM singlet complex scalar flavon, θ . Its charge under $U(1)_F$ is $F_\theta \neq 0$, and we denote its vacuum expectation value (VEV) by v_F . We denote the original $U(1)_F$ gauge boson by X , reserving the name Z' for the physical boson (which is a mass eigenstate). The original Z boson of the SM mixes with this X boson to a small degree because the neutral component of H , which achieves a VEV v , has both $U(1)_F$ and $SU(2) \times U(1)_Y$ quantum numbers.⁸ Following refs. [38, 48] (which examined some $Z - Z'$ mixing constraints in different $SM \times U(1)$ models), the relevant mass terms come from the kinetic terms of the scalar fields H and θ :

$$\mathcal{L}_{H\theta K} = (D^\mu H)^\dagger (D_\mu H) + (D^\mu \theta)^* (D_\mu \theta), \quad (2.2)$$

where the covariant derivatives are

$$D_\mu H = \partial_\mu H - i \frac{g}{2} \left(\tau^a W_\mu^a - \frac{g'}{g} B_\mu - \frac{g_F}{g} X_\mu \right) H, \quad D_\mu \theta = (\partial_\mu - i F_\theta g_F X_\mu) \theta, \quad (2.3)$$

where, as usual, g and g' denote the gauge couplings for $SU(2)_L$ and $U(1)_Y$ respectively, and g_F denotes the gauge coupling for $U(1)_F$.

Expanding the scalar fields about their VEVs in eq. (2.2), *viz.* $H = (0, v + h(x))^T / \sqrt{2}$ and $\theta = (v_F + s(x)) / \sqrt{2}$, leads to mass terms for the neutral gauge bosons of the form $\mathcal{L}_{N,\text{mass}} = \frac{1}{2} \mathbf{A}'_\mu{}^T \mathcal{M}_N^2 \mathbf{A}'_\mu$, where $\mathbf{A}'_\mu = (B_\mu, W_\mu^3, X_\mu)^T$,⁹ and the mass matrix is

$$\mathcal{M}_N^2 = \frac{v^2}{4} \begin{pmatrix} g'^2 & -gg' & g'g_F \\ -gg' & g^2 & -gg_F \\ g'g_F & -gg_F & g_F^2 (1 + 4F_\theta^2 r^2) \end{pmatrix}, \quad (2.4)$$

where $r \equiv v_F/v \gg 1$ is the ratio of the VEVs. One can check that the determinant of \mathcal{M}_N^2 vanishes, hence there remains a massless photon. Writing $\mathcal{L}_{N,\text{mass}} =$

⁸We assume that the kinetic term for the gauge fields themselves, which should *a priori* include an off-diagonal term mixing the two $U(1)$ gauge fields, has already been diagonalised.

⁹Here, the prime on \mathbf{A}'_μ denotes that the gauge fields are in the $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_F$ eigenbasis.

$\frac{1}{2}\mathbf{A}'_\mu{}^T O O^T \mathcal{M}_N^2 O O^T \mathbf{A}'_\mu$, where O is an orthogonal matrix such that $O^T \mathcal{M}_N^2 O = \text{diag}(0, M_Z^2, M_{Z'}^2)$, we define the mass basis of physical neutral gauge bosons via $(A_\mu, Z_\mu, Z'_\mu)^T \equiv \mathbf{A}_\mu = O^T \mathbf{A}'_\mu$. The orthogonal matrix O can be written

$$O = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \cos \alpha_z & \sin \theta_w \sin \alpha_z \\ \sin \theta_w & \cos \theta_w \cos \alpha_z & -\cos \theta_w \sin \alpha_z \\ 0 & \sin \alpha_z & \cos \alpha_z \end{pmatrix}, \quad (2.5)$$

where θ_w is the Weinberg angle (such that $\tan \theta_w = g'/g$), and the $Z - Z'$ mixing angle α_z is related to the masses of the Z and Z' via the equation

$$(M_{Z'}^2 - M_Z^2) \sin 2\alpha_z = \frac{gg_F v^2}{2 \cos \theta_w}.$$

We shall now assume that the Z' is much heavier than the Z boson, such that the mixing between them is small. In the (consistent) limit that $M_Z/M'_Z \ll 1$ and $\sin \alpha_z \ll 1$, the masses of the heavy neutral gauge bosons are given by

$$M_Z \approx \frac{M_W}{\cos \theta_w} = M_W \frac{\sqrt{g^2 + g'^2}}{g}, \quad M_{Z'} \approx M_W \frac{g_F \sqrt{1 + 4F_\theta^2 r^2}}{g}, \quad (2.6)$$

where $M_W = gv/2$, and the mixing angle is

$$\sin \alpha_z \approx \frac{g_F}{\sqrt{g^2 + g'^2}} \left(\frac{M_Z}{M'_Z} \right)^2. \quad (2.7)$$

Recall that the ratio of VEVs $r = v_F/v$ is much larger than one, such that the Z' is indeed expected to be much heavier than the electroweak gauge bosons of the SM.

From the relation $\mathbf{A}_\mu = O^T \mathbf{A}'_\mu$, and eq. (2.5), one deduces that the photon remains the same linear combination of B and W^3 as in the SM. The physical Z boson, however, now contains a small admixture of the X field:

$$Z_\mu = \cos \alpha_z (-\sin \theta_w B_\mu + \cos \theta_w W_\mu^3) + \sin \alpha_z X_\mu, \quad (2.8)$$

and so will inherit small flavour-changing corrections to its fermionic couplings. Thus, we must take the Z boson mediated contributions into account when calculating flavour violating effective operators. The $Z - Z'$ mixing must be consistent with a constraint from LEP, as we shall see in section 3.1.

2.2 Z' couplings to fermions

We begin with the couplings of the $U(1)_F$ gauge boson X_μ to fermions in the Lagrangian in the weak eigenbasis

$$\mathcal{L}_{X\psi} = g_F \left(\frac{1}{6} \overline{Q'_{3L}} \gamma^\rho Q'_{3L} - \frac{1}{2} \overline{L'_{3L}} \gamma^\rho L'_{3L} - \overline{e'_{3R}} \gamma^\rho e'_{3R} + \frac{2}{3} \overline{u'_{3R}} \gamma^\rho u'_{3R} - \frac{1}{3} \overline{d'_{3R}} \gamma^\rho d'_{3R} \right) X_\rho, \quad (2.9)$$

where g_F is the $U(1)_F$ gauge coupling. We saw in section 2.1 that the $U(1)_F$ gauge boson X is equal to the physical heavy gauge boson Z' (which is a mass eigenstate) up to a small correction. In order to calculate the effects on the mass eigenbasis fields, we must provide the connection to the weak eigenbasis for the fermions: the details and conventions are set out in appendix A. In the mass basis, using eqs. (2.5), (2.7), and (A.3), eq. (2.9) becomes

$$\begin{aligned} \mathcal{L}_{X\psi} = g_F \left(\frac{1}{6} \overline{\mathbf{u}}_{\mathbf{L}} \Lambda^{(u_L)} \gamma^\rho \mathbf{u}_{\mathbf{L}} + \frac{1}{6} \overline{\mathbf{d}}_{\mathbf{L}} \Lambda^{(d_L)} \gamma^\rho \mathbf{d}_{\mathbf{L}} - \frac{1}{2} \overline{\mathbf{n}}_{\mathbf{L}} \Lambda^{(n_L)} \gamma^\rho \mathbf{n}_{\mathbf{L}} - \frac{1}{2} \overline{\mathbf{e}}_{\mathbf{L}} \Lambda^{(e_L)} \gamma^\rho \mathbf{e}_{\mathbf{L}} \right. \\ \left. + \frac{2}{3} \overline{\mathbf{u}}_{\mathbf{R}} \Lambda^{(u_R)} \gamma^\rho \mathbf{u}_{\mathbf{R}} - \frac{1}{3} \overline{\mathbf{d}}_{\mathbf{R}} \Lambda^{(d_R)} \gamma^\rho \mathbf{d}_{\mathbf{R}} - \overline{\mathbf{e}}_{\mathbf{R}} \Lambda^{(e_R)} \gamma^\rho \mathbf{e}_{\mathbf{R}} \right) Z'_\rho, \end{aligned} \quad (2.10)$$

where each of the couplings is missing small $\mathcal{O}(M_Z^2/M_{Z'}^2)$ terms induced by $Z-Z'$ mixing, and we have defined the 3 by 3 dimensionless Hermitian coupling matrices

$$\Lambda^{(I)} \equiv V_I^\dagger \xi V_I, \quad (2.11)$$

where $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$ and

$$\xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.12)$$

This completes our definition of the Third Family Hypercharge Model. Provided that $(V_{e_L})_{23} \neq 0$ and $(V_{d_L})_{23} \neq 0$, eq. (2.10) contains couplings to $\overline{b}_L s_L + \text{H.c.}$ and $\overline{\mu}_L \mu_L$, and so is a promising model for explaining the discrepancies between the measurements of $R_{K^{(*)}}$ and their SM predictions.

2.3 Example case

In order to identify the couplings of the model further, we need to specify the mixing matrices V_I . However, at this coarse level of model building, we do not have an explicit model for them. We now make a number of (fairly strong) assumptions in order to specify a model, but we emphasise that these just provide an example case of the model for further study.

We know that we require a coupling of the Z' to $\mu^+ \mu^-$ and to $\overline{b}s$, in order to produce the effective operators in eq. (1.2). The existence of these couplings implies that V_{d_L} and V_{e_L} should contain some mixing between the third and second generations. For now, we will take the limiting case that

$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{sb} & -\sin \theta_{sb} \\ 0 & \sin \theta_{sb} & \cos \theta_{sb} \end{pmatrix} \quad \text{and} \quad V_{e_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (2.13)$$

where we expect $|\sin \theta_{sb}| \sim \mathcal{O}(|V_{ts}|)$. Sometimes, we shall exemplify with $\sin \theta_{sb} = |V_{ts}| = 0.04$ (when we shall explicitly state it). Eq. (2.13) implies that there are no tree-level flavour changing currents between the first two generations of down quark, circumventing

strong $K^0 - \bar{K}^0$ mixing constraints.¹⁰ From eq. (A.4), we require $V_{u_L} = V_{d_L} V^\dagger$. So as not to produce Z' couplings to $\bar{b}_R s_R + \text{H.c.}$ (such couplings dominating is disfavoured by fits to B -data [4]), we set $V_{d_R} = 1$. For simplicity and definiteness, we also set $V_{u_R} = 1$. We have chosen V_{e_L} in eq. (2.13) to transfer the Z' coupling from the third family entirely into the second in the (left-handed) charged leptons, so as to induce the $\bar{\mu}_L \mu_L$ coupling to the Z' . This is really a constraint upon the charged lepton Yukawa matrix, which, up to small corrections, should then be

$$Y_E = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_\tau \\ 0 & x & 0 \end{pmatrix}, \quad (2.14)$$

where x is a Yukawa coupling contributing to the muon Dirac mass after electroweak symmetry breaking. In other words, to realise this example case the 33 element of Y_E must be suppressed relative to the naïve expectation, which presents a requirement on more detailed model building. Note that since $Y_\tau \sim \mathcal{O}(10^{-2})$ anyway, the non-zero Yukawa couplings in eq. (2.14) can still plausibly result from non-renormalisable operators as required from our charge assignment in table 1.¹¹ In this particular form of the example case therefore, the Third Family Hypercharge model *per se* only explains the hierarchical heaviness of the third family of quark: more detailed model building would be needed to understand that of the leptons.

Eq. (2.13) should be understood as a straightforward limiting case which fits the data at present (as we discuss in detail in section 3): it allows for a large coupling of the Z' to muons, but kills Z' couplings to left-handed electrons which have strong constraints from LEP. Furthermore, with this choice there is no Z' coupling to left-handed $\mu^\pm \tau^\mp$ pairs, which means this example case is automatically consistent with very strong constraints from the measurement of the $\tau \rightarrow \mu \mu \mu$ branching ratio [49]. Simplicity also motivates us to set $V_{e_R} = 1$, but eq. (A.4) implies that we must set $V_{\nu_L} = V_{e_L} U^\dagger$.

Substituting these matrices into eq. (2.10), we obtain

$$\begin{aligned} \mathcal{L}_{X\psi} = g_F & \left(\frac{1}{6} \bar{\mathbf{u}}_L \Lambda^{(u_L)} \gamma^\rho \mathbf{u}_L + \frac{1}{6} \bar{\mathbf{d}}_L \Lambda^{(d_L)} \gamma^\rho \mathbf{d}_L - \frac{1}{2} \bar{\mathbf{n}}_L \Lambda^{(n_L)} \gamma^\rho \mathbf{n}_L - \frac{1}{2} \bar{\mu}_L \gamma^\rho \mu_L \right. \\ & \left. + \frac{2}{3} \bar{t}_R \gamma^\rho t_R - \frac{1}{3} \bar{b}_R \gamma^\rho b_R - \bar{\tau}_R \gamma^\rho \tau_R \right) Z'_\rho, \end{aligned} \quad (2.15)$$

where $\Lambda^{(u_L)} = V V_{d_L}^\dagger \xi V_{d_L} V^\dagger$, $\Lambda^{(n_L)} = U V_{e_L}^\dagger \xi V_{e_L} U^\dagger$, and

$$\Lambda^{(d_L)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2 \theta_{sb} & \frac{1}{2} \sin 2\theta_{sb} \\ 0 & \frac{1}{2} \sin 2\theta_{sb} & \cos^2 \theta_{sb} \end{pmatrix}. \quad (2.16)$$

¹⁰Promoting the zeroes in V_{d_L} to CKM-suppressed elements does provide constraints but does not rule all of the otherwise viable parameter space of the model out.

¹¹Indeed, within a Froggatt-Nielsen setup with flavon charge $F_\theta = \pm 1/2$, one would expect the coupling Y_τ to be hierarchically larger than x , thereby being consistent with the charged lepton mass hierarchy (provided, of course, that one can suppress the 33 element of Y_E with more detailed model building).

From these,¹² we read off the couplings relevant for causing the recent neutral current lepton flavour non-universality measurements in B -decays,

$$\mathcal{L}_{X\psi} = \left(\frac{g_F}{12} \sin 2\theta_{sb} \bar{s} \gamma^\rho P_L b - \frac{g_F}{2} \bar{\mu} \gamma^\rho P_L \mu + \text{H.c.} \right) Z'_\rho + \dots \quad (2.17)$$

Eq. (2.17) is a promising operator for explaining $R_{K^{(*)}}$: it only has left-handed currents between $\bar{b}s$ and $\bar{\mu}\mu$. Also, the Z' coupling to $\bar{s}b$ is suppressed by $(\sin 2\theta_{sb})/6$ compared to its coupling to muons. This helps explain why the model does not induce a large new physics contribution to $B_s - \bar{B}_s$ mixing that would be incompatible with measurements, but can still explain $R_{K^{(*)}}$, which we show in the next section where we examine the phenomenology of the example case.

As one can see from eq. (2.15), the Z' also has couplings to all flavours of left-handed (LH) up-type quarks, and all flavours of LH neutrinos. These couplings result in various additional constraints on the model (and predictions of the model), both through direct couplings to the Z' boson and via modified Z couplings due to the $Z-Z'$ mixing. Explicitly, $\Lambda^{(u_L)}$ has matrix elements given by

$$\Lambda_{ij}^{(u_L)} = \cos^2 \theta_{sb} V_{ib} V_{jb}^* + \sin^2 \theta_{sb} V_{is} V_{js}^* + \frac{1}{2} \sin 2\theta_{sb} (V_{is} V_{jb}^* + V_{ib} V_{js}^*), \quad (2.18)$$

where the indices i and j here run over the up-type flavours u , c , and t . Numerically, for $(\sin 2\theta_{bs})/2 = 0.04$, the magnitudes of these couplings are $g_F/6$ multiplied by

$$\Lambda^{(u_L)} = \begin{pmatrix} 0.0002 & 0.001 & 0.012 \\ 0.001 & 0.006 & 0.079 \\ 0.012 & 0.079 & 0.995 \end{pmatrix}. \quad (2.19)$$

3 Phenomenology of example case

We now examine the phenomenology of our Third Family Hypercharge Model example case, beginning with constraints, then providing predictions in terms of Z' width and branching ratios, and predictions for B meson decays.

3.1 Constraints

We expect the strongest constraints upon our model to come from fitting $R_{K^{(*)}}$, $B_s - \bar{B}_s$ mixing measurements, and from the measurements of Z boson couplings to the first two generations of leptons derived from LEP. There may be additional constraints coming from other electroweak measurements: these may even provide an opportunity for our model to better fit the forward-backward asymmetry of b -quarks measured by LEP, which differs to the SM fit by some $\sim 2.3\sigma$ [49]. Such a study would require a combined fit to electroweak data and is outside the scope of the present paper. We leave it for future work, focusing now on the other constraints in turn.

¹²Some aspects of Z' couplings of the example case are somewhat similar to an ansatz proposed in ref. [50], which examined phenomenological bounds on them.

3.1.1 Neutral current B meson measurements

A fit [4] to $R_{K^{(*)}}$ and selected other ‘clean’ (i.e. observables with particularly low theoretical uncertainties) B -observables found that the couplings and mass of Z' particles are constrained to be

$$g_{sb}g_{\mu\mu} = -x \left(\frac{M_{Z'}}{31 \text{ TeV}} \right)^2, \quad (3.1)$$

where $x = 1.00 \pm 0.25$ from the fit and the relevant Z' couplings are defined to be¹³

$$\mathcal{L}_{Z'f} = \left(g_{sb} Z'_\rho \bar{s}_L \gamma^\rho b_L + \text{H.c.} \right) + g_{\mu\mu} Z'_\rho \bar{\mu}_L \gamma^\rho \mu_L + \dots \quad (3.2)$$

From eq. (2.17) we identify $g_{sb} = g_F(\sin 2\theta_{sb})/12$ and $g_{\mu\mu} = -g_F/2$ in our example case. We can then match eq. (3.1) on to a constraint on g_F and $M_{Z'}$:

$$g_F^2 = x \frac{24}{\sin 2\theta_{sb}} \left(\frac{M_{Z'}}{31 \text{ TeV}} \right)^2 = x \left(\frac{M_{Z'}}{1.79 \text{ TeV}} \right)^2 \frac{0.04}{\frac{1}{2} \sin 2\theta_{sb}}. \quad (3.3)$$

This translates to the bounds

$$\frac{M_{Z'}}{2.53 \text{ TeV}} \sqrt{\frac{0.04}{\frac{1}{2} \sin 2\theta_{sb}}} < g_F < \frac{M_{Z'}}{1.46 \text{ TeV}} \sqrt{\frac{0.04}{\frac{1}{2} \sin 2\theta_{sb}}} \quad (3.4)$$

at the 95% Confidence Level (CL).

3.1.2 Neutral meson mixing

The Z' coupling to $b\bar{s}$ which is needed to fit $R_{K^{(*)}}$ also results in a tree level contribution to $B_s - \bar{B}_s$ mixing (which, in the SM, arises from box diagrams and so is loop-suppressed). We adapt the bound on $B_s - \bar{B}_s$ mixing from refs. [4, 51], using the 2σ 2016 FLAG average on the hadronic form factor f_{B_s} and the bag parameter B_{B_s} . The resulting bound is equivalent to¹⁴

$$\frac{g_F}{12} \sin 2\theta_{sb} < \frac{M_{Z'}}{148 \text{ TeV}} \Rightarrow g_F < \left(\frac{M_{Z'}}{1.0 \text{ TeV}} \right) \left(\frac{0.04}{\frac{1}{2} \sin 2\theta_{sb}} \right). \quad (3.5)$$

In addition to the Z' contribution, there is also a tree level contribution to $B_s - \bar{B}_s$ mixing from Z boson exchange in our model, due to the $Z - Z'$ mixing. However, this contribution is suppressed with respect to the Z' contribution by $\mathcal{O}(M_Z/M_{Z'})^2$ and so we neglect it. Flavour-changing couplings of the Z' to the down-type quarks (induced by promoting some of the zeroes in eq. (2.15) to finite quantities) would also induce corrections beyond the SM to the mixings of other neutral mesons, for example $K^0 - \bar{K}^0$ mixing or $B_d - \bar{B}_d$ mixing [53]. These would induce additional constraints on the model.

¹³We note that we predict a tree-level Z boson contribution to the $(\bar{b}_L s_L)(\bar{\mu}_L \mu_L)$ operator in this model but since, to leading order in $(M_Z^2/M_{Z'}^2)$, there is an identical Z contribution to $(\bar{b}_L s_L)(\bar{e}_L e_L)$, it cancels in R_K and R_{K^*} .

¹⁴A recent determination of f_{B_s} and B_{B_s} by the Fermilab/MILC collaboration [51] is in tension with other previous estimates. When used to extract the Standard Model prediction of the $B_s - \bar{B}_s$ mixing parameter Δm_s , it is also in tension with the experimental determination. However, were we to use these determinations, stronger bounds on new physics would follow [52], implying $|g_{sb}| \lesssim M_{Z'}/600 \text{ TeV}$.

3.1.3 Lepton flavour universality of the Z boson

In the SM, the Z boson is a linear combination of B and W^3 , *viz.* $Z_\mu^{\text{SM}} = \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu$, whereas in the Third Family Hypercharge Model the Z contains a small admixture of the $U(1)_F$ gauge boson X , as in eq. (2.8). Since the fermion couplings to X are flavour-dependent, this introduces non-universality to the leptonic decays of the Z , which are constrained by the LEP measurement [49]:

$$R_{\text{LEP}} = 0.999 \pm 0.003, \quad R \equiv \frac{\Gamma(Z \rightarrow e^+ e^-)}{\Gamma(Z \rightarrow \mu^+ \mu^-)}. \quad (3.6)$$

In the Third Family Hypercharge Model, the partial width for $Z \rightarrow e^+ e^-$ is unchanged from the SM, to leading order in α_z , because the Z' does not couple to (left-handed or right-handed) electrons.¹⁵ In contrast, the partial width for $Z \rightarrow \mu^+ \mu^-$ is modified at leading order, because of the X coupling to left-handed muon pairs.

Within the Third Family Hypercharge Model, the ratio of partial widths is

$$R_{\text{model}} = \frac{|g_Z^{eLeL}|^2 + |g_Z^{eReR}|^2}{|g_Z^{\mu L \mu L}|^2 + |g_Z^{\mu R \mu R}|^2}, \quad (3.7)$$

where g_Z^{ff} is the coupling of the physical Z boson to the fermion pair $f\bar{f}$. One can obtain the couplings g_Z^{ff} by first writing down the terms in the Lagrangian which couple the charged leptons to the neutral bosons B , W^3 , and X :

$$\begin{aligned} \mathcal{L}_{lZ'} = & \bar{e}_L \left(-\frac{1}{2} g W^3 - \frac{1}{2} g' \not{B} \right) e_L + \bar{\mu}_L \left(-\frac{1}{2} g W^3 - \frac{1}{2} g' \not{B} - \frac{1}{2} g_F \not{X} \right) \mu_L + \\ & \bar{\tau}_L \left(-\frac{1}{2} g W^3 - \frac{1}{2} g' \not{B} \right) \tau_L + \bar{\mathbf{e}}_R (-g' \not{B}) \mathbf{e}_R + \bar{\tau}_R (-g_F \not{X}) \tau_R, \end{aligned} \quad (3.8)$$

and then inserting $\mathbf{A}_\mu' = O \mathbf{A}_\mu$ (where O is given in eq. (2.5)) to rotate to the mass basis. To leading order in $\sin \alpha_z$, we find:

$$\begin{aligned} g_Z^{eLeL} &= -\frac{1}{2} g \cos \theta_w + \frac{1}{2} g' \sin \theta_w, \\ g_Z^{\mu L \mu L} &= -\frac{1}{2} g \cos \theta_w + \frac{1}{2} g' \sin \theta_w - \frac{1}{2} g_F \sin \alpha_z, \\ g_Z^{eReR} &= g_Z^{\mu R \mu R} = g' \sin \theta_w. \end{aligned} \quad (3.9)$$

The SM prediction (i.e. $R = 1$) is recovered by taking α_z to zero. Within the Third Family Hypercharge Model, we may expand R_{model} to leading order in $\sin \alpha_z$:

$$R_{\text{model}} = 1 - \frac{2g_F(g \cos \theta_w - g' \sin \theta_w) \sin \alpha_z}{(g \cos \theta_w - g' \sin \theta_w)^2 + 4g'^2 \sin^2 \theta_w} = 1 - 4.2g_F^2 \left(\frac{M_Z}{M_{Z'}} \right)^2, \quad (3.10)$$

after substituting in eq. (2.7) for $\sin \alpha_z$, and the central experimental values $g = 0.64$ and $g' = 0.34$. Comparison with the lower LEP limit, at the 95% CL, yields the Z -LEP lepton flavour universality constraint (LEP LFU) from eq. (3.6):

$$g_F^2 \left(\frac{M_Z}{M_{Z'}} \right)^2 < 0.0017 \Rightarrow g_F < \frac{M_{Z'}}{2.2 \text{ TeV}}. \quad (3.11)$$

¹⁵There is of course a reduction in the Z boson couplings to electrons arising from the factor of $\cos \alpha_z$ in eq. (2.8), however this shift is of order α_z^2 and is therefore subleading.

Other constraints from LEP measurements of fermionic couplings to Z bosons (for example $b\bar{b}$) are weaker than this. We see that LEP LFU yields a tighter constraint than the one from $B_s - \bar{B}_s$ in eq. (3.5) for $\frac{1}{2} \sin 2\theta_{sb} \lesssim 0.08$.

3.1.4 $t \rightarrow Zq$ decays

One might worry that in our example case we have introduced various tree level flavour changing neutral current interactions involving the top quark and the lighter up-type quarks u and c . However, it turns out that constraints on our model from flavour-changing tZ interactions are very weak, as we now summarise for completeness. In the example case of the Third Family Hypercharge Model, the Lagrangian in eq. (2.15) contains the interactions¹⁶

$$\mathcal{L}_{Xtq} = \frac{g_F}{6} \left(\Lambda_{23}^{(u_L)} \bar{c} \gamma^\rho P_L t + \Lambda_{13}^{(u_L)} \bar{u} \gamma^\rho P_L t + \text{H.c.} \right) X_\rho, \quad (3.12)$$

(where $\Lambda_{23}^{(u_L)} \approx V_{cb} V_{tb}^* + \frac{1}{2} \sin 2\theta_{sb} V_{cs} V_{tb}^*$ and $\Lambda_{13}^{(u_L)} \approx V_{ub} V_{tb}^* + \frac{1}{2} \sin 2\theta_{sb} V_{us} V_{tb}^*$), facilitating the decays $t \rightarrow Zu$ and $t \rightarrow Zc$ at tree-level via the $Z - Z'$ mixing. In the example case, the branching ratio for $t \rightarrow Zc$ is predicted to be

$$\begin{aligned} BR(t \rightarrow Zc) &= \frac{g_F^2 \Lambda_{23}^{(u_L)2} f(M_Z, M_W, M_t) \sin^2 \alpha_z}{18g^2 |V_{tb}|^2} BR(t \rightarrow Wb) \\ &= 1.1 \times 10^{-3} g_F^4 \left(\frac{M_Z}{M_{Z'}} \right)^4 \left(\frac{|V_{cb} V_{tb}^* + \frac{1}{2} \sin 2\theta_{sb} V_{cs} V_{tb}^*|^2}{0.0062} \right), \end{aligned} \quad (3.13)$$

where $f(M_Z, M_W, M_t)$ is a kinematical factor,¹⁷ and we have assumed the top's branching ratio to Wb is unity and neglected the masses of the bottom and charm quarks. Using the CMS bound from the 8 TeV LHC data, $BR(t \rightarrow Zc) < 4.9 \times 10^{-4}$ at 95% CL [54], yields the weak constraint $g_F < M_{Z'}/(0.1 \text{ TeV})$ when $\frac{1}{2} \sin 2\theta_{sb} = 0.04$. Performing a similar calculation for $t \rightarrow Zu$ using the CMS 8 TeV 95% CL bound, $BR(t \rightarrow Zu) < 2.2 \times 10^{-4}$, yields a yet weaker constraint on $g_F/M_{Z'}$.

3.1.5 Combination of constraints

We summarise the constraints on our example case in figure 1. We see in the LH panel the white region of parameter space, which fits $R_{K^{(*)}}$ whilst remaining on the right side of the LEP LFU and $B_s - \bar{B}_s$ mixing constraints. It is encouraging that our example case can satisfy all bounds when the fermion mixings are tightly constrained by rather simple and definite choices. The central value of the fit to clean B -physics observables can be achieved for $0.13 \geq \theta_{sb} \geq 0.06$, as shown by the blue line in the LH panel, and the constraints. If we were to choose more general fermion mixing matrices, we might widen the allowed region. For now, we leave the example case as an existence proof.

¹⁶As can be seen from eq. (2.19), the interactions involving only u and c are extremely suppressed, because the X couplings are “mixed in” from the third family in the Third Family Hypercharge Model.

¹⁷Explicitly,

$$f(M_Z, M_W, M_t) = \frac{M_W^2}{M_Z^2} \left(1 - \frac{M_Z^2}{M_t^2} \right)^2 \left(1 + \frac{2M_Z^2}{M_t^2} \right) \left(1 - \frac{M_W^2}{M_t^2} \right)^{-2} \left(1 + \frac{2M_W^2}{M_t^2} \right)^{-1},$$

where we have neglected the masses of the bottom and charm quarks. We have used $M_W = 80.4 \text{ GeV}$, $M_Z = 91.19 \text{ GeV}$ and $M_t = 173 \text{ GeV}$ in evaluating the right-hand side of eq. (3.13).

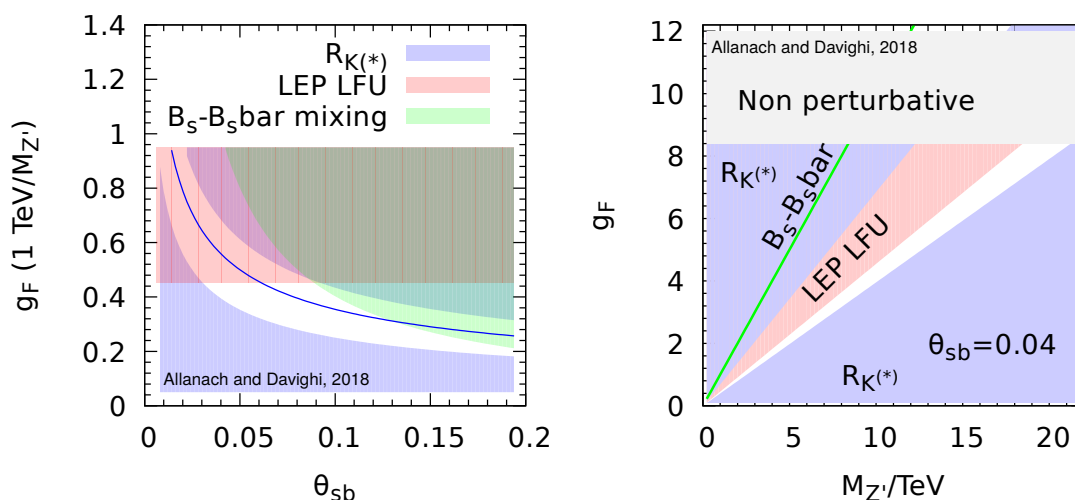


Figure 1. Summary of 95% CL constraints upon the Third Family Hypercharge Model example case. Each constraint excludes the labelled coloured area, leaving an allowed white region. $R_{K^{(*)}}$ refers to a fit to ‘clean’ B –physics observables [4], including to R_K and $R_{K^{(*)}}$. The blue curve in the LH plot is where the fit is central. ‘LEP LFU’ shows the LEP lepton flavour universality constraint in eq. (3.11), whereas ‘ $B_s - \bar{B}_s$ ’ shows the constraint in eq. (3.5). In the right-hand plot, we show constraints in the $g_F - M_{Z'}$ plane for the choice $\theta_{sb} = 0.04$. In this plot, the region ruled out by the $B_s - \bar{B}_s$ mixing constraint is *above* the line. The ‘Non perturbative’ region is defined as the region where $\Gamma_{Z'} > M_{Z'}$ and is estimated in section 3.2.

3.2 Predictions

Here, we sketch the main experimental predictions of the model. The Z' particle may be able to be produced and measured [47] either at the LHC, the high luminosity run of the LHC, or the high energy run of the LHC or a 100 TeV future circular pp collider in order to provide a direct test of the Third Family Hypercharge Model, and hence of the mechanism that generates the hierarchically heavy third family of charged fermions. The classic signature of such a Z' would be a bump in the muon anti-muon pair production cross-section [47]. Current searches by LHC general purpose experiments have so far found no such bump, for example excluding $M_{Z'} < 4.0, 4.5 \text{ TeV}$ for Z' models that couple identically to the SM Z boson [55, 56]. The SM Z boson has sizeable couplings to up and down quarks, whereas in our example case the Z' has only very small quark couplings, except for the third generation: the bounds from such direct searches are then very much less sensitive than the 4.0-4.5 TeV masses quoted. We leave the re-casting of LHC bounds for our model to future work.

Assuming that $M_{Z'} \gg 2m_t$ (since otherwise it would likely have been discovered already), we may neglect fermion masses in its decays. The partial width of Z' into a massless fermion f_i and anti-fermion \bar{f}_j is $\Gamma_{f_i f_j} = C/(24\pi)|g_{ij}|^2 M_{Z'}$, where g_{ij} is the coupling of the Z' to $f_i \bar{f}_j$, and $C = 3$ if the fermions are coloured but $C = 1$ otherwise. Z' has a tree-level coupling to HZ in our model, which stems from giving the Higgs a $U(1)_F$ charge. Specifically, upon rotation to the mass basis of the neutral gauge bosons, as in section 2.1,

Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.42	$b\bar{b}$	0.12	$\nu\bar{\nu}'$	0.08
$\mu^+\mu^-$	0.08	$\tau^+\tau^-$	0.30	other $f_i f_j$	$\sim \mathcal{O}(10^{-4})$

Table 2. Z' branching ratios (BRs) in the Third Family Hypercharge Model example case. We have neglected fermion masses and lumped all flavours of neutrino into ν, ν' . The BRs to other fermion pairs are highly suppressed; for example, the next largest BR is to $t\bar{c}$ pairs, for which the BR is $\sim \mathcal{O}(10^{-4})$.

one finds the Lagrangian terms in eq. (2.2) contain a piece $\mathcal{L}_{H\theta K} \supset \lambda h Z'_\mu Z^\mu$, where we calculate the coefficient to be $\lambda = -2g_F^2 F_\theta^2 (v_F^2/v) \sin \alpha_z \cos \alpha_z$. Thus, the partial width of Z' into HZ is $\propto M_{Z'}/(16\pi^2)\mathcal{O}(M_Z/M_{Z'})^2$ and so is negligible compared to decays into fermions. Neglecting this mode and neglecting fermion masses, and working from the weak eigenbasis couplings in eq. (2.9), we obtain a total Third Family Hypercharge Model Z' width of

$$\Gamma_{Z'} = \frac{5g_F^2 M_{Z'}}{36\pi^2}, \quad (3.14)$$

where the branching ratio into quarks is 11/20 and the branching ratio into leptons is 9/20. The Z' total width is equal to its mass when $g_F = 6\pi/\sqrt{5} = 8.4$. For g_F values of this size and above, the model enters a non-perturbative régime, which is indicated in figure 1. In order to further specify the Z' branching ratios into different flavours of quark and lepton, one must specify the V_I mixing matrices.

For the example case that we have detailed in section 2.3, the branching ratios are as in table 2, where we have taken the central values of CKM and PMNS matrix elements from the Particle Data Group [49]. We see from table 2 that the example case predicts that a bump in the $\mu^+\mu^-$ invariant mass spectrum at $M_{Z'}$ is suppressed by the $\approx 8\%$ branching ratio. Other promising discovery modes are likely to then be into boosted top pairs and tau pairs (for which the branching ratios are bigger because of the larger couplings of the Z' to *right-handed* taus). It will be interesting to compare sensitivities to the different channels quantitatively, in the future, and to estimate the sensitivities to measuring the top and tau polarisations, which are different to those produced by Z bosons.

The example case predicts a non-SM contribution to $BR(B \rightarrow K^{(*)}\tau^+\tau^-)$. As such, it follows some of the expectations from ref. [57]. Identifying τ particles resulting from B decays is difficult experimentally, and so we may have to wait for future LHC and Belle II runs before there is sufficient sensitivity to these modes. Near future prospects for improving and checking the measurements of R_K and $R_K^{(*)}$ remain very good, however [58, 59].

We have left the study of the Higgs potential including the flavon θ for the future. However, a gauge invariant term in the potential $\lambda_{\theta H}|\theta|^2|H^2|$ is present at the renormalisable level, where $\lambda_{\theta H}$ is a dimensionless coupling. Since θ and H both acquire VEVs, this term will induce flavon-Higgs mixing. This could then affect Higgs couplings, particularly to taus, tops and bottom quarks. It is clear that one can remove these effects in the limit $\lambda_{\theta H} \rightarrow 0$, but applying current experimental bounds on Higgs branching ratios would provide an upper limit on $\lambda_{\theta H}$.

In the present paper, we have focused on the tree-level phenomenology. There are small effects at the one-loop level, for example due to $U(1)_Y - U(1)_F$ mixing (and indeed from Higgs-flavon mixing, even in the $\lambda_{\theta H} \rightarrow 0$ limit), that are beyond the scope of our paper but may be interesting to address nonetheless.

4 Conclusions

We have constructed a model with a gauged flavoured $U(1)_F$ group. Once it has been spontaneously broken via

$$SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_F \xrightarrow{\theta} SU(3) \times SU(2)_L \times U(1)_Y \xrightarrow{H} SU(3) \times U(1)_{em},$$

our model explains some coarse features of fermion masses and mixings and it provides an explanation for inferred non-SM contributions to $R_{K^{(*)}}$. In particular, the hierarchical heaviness of the third family of charged fermions is predicted. We imagine that small non-renormalisable operators will induce quark mixing and masses for the lighter two families. CKM mixing will then be predicted to be small. PMNS mixing, however, is not necessarily predicted to be small. For example, in our example case we induce a large 23 PMNS mixing by requiring the 33 element of the charged lepton Yukawa be suppressed. More generally, in any implementation of the Third Family Hypercharge Model, large PMNS mixing can result from the neutrino sector, which we shall now discuss.

The only dimension 5 term allowed by the $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_F$ gauge symmetry in the fermion sector is $\mathcal{L}_{SS} = \frac{1}{2M}(L'_3{}^T H^c)(L'_3 H^c)$, leading to a third family neutrino mass after H develops a VEV. The first two neutrino masses are not present at this order in the effective field theory expansion, being banned by $U(1)_F$, and so one might *prima facie* expect the model to predict a normal hierarchy and small mixing in the neutrino sector. However, once a more detailed model for non-renormalisable terms is built (for example by including right-handed neutrinos), we may expect the effective dimension 5 terms to instead be $1/2 \sum_{ij}(L'_i H^c)(M^{-1})_{ij}(L'_j H^c)$, where now $(M^{-1})_{ij}$ may well have a non-trivial structure depending on details of the model. If some of the elements of $(M^{-1})_{ij}$ may be predicted to be of the same order of magnitude, then an explanation for large PMNS mixing can result. By extending the model with right-handed neutrinos in such a way, and by a careful assignment of charges and implementation of the Froggatt-Nielsen mechanism, we would like to provide a full account of *both* the mass hierarchies and the mixing angles in all the SM fermions. We intend to explore such avenues in the future.

We emphasise here that the example case presented in section 2.3 contains some very strong assumptions, where various limits are taken for definiteness. One would ideally want to derive the structure from the Froggatt-Nielsen (or similar) mechanism, and thereby to develop a more refined example case for study. However, the example case which we have set up in this paper helps eke out phenomenological predictions in a particular limit. Suggestions for future measurements issuing from this include bounding B decays to $K^{(*)}\tau^+\tau^-$, searching for the Z' in boosted top pairs, di-taus and di-muons, and high luminosity LHC searches for e.g. $t \rightarrow Zc$ decays.

One can imagine variants of the model. One such variant would be to make the Z' only couple to the μ flavour of charged lepton, by switching the second family lepton hypercharges with those of the third family: $F_{L'_3} = F_{e_{R'_3}} = 0$, $F_{L'_2} = -1/2$, $F_{e_{R'_2}} = -1$, with all other charges as in table 1.¹⁸ In this case, the tau Yukawa coupling would be absent from eq. (2.1), and so it would need to be produced by a non-renormalisable operator with effective coefficient $\approx \mathcal{O}(10^{-2})$. On the other hand, the muon Yukawa coupling *would* be present at the renormalisable level and would need to be set to be fairly small: $\mathcal{O}(m_\mu/m_t) \sim 10^{-3}$. In this case, one could fix $V_{eL} = 1$ meaning that *all* of the PMNS mixing would come from the neutrinos, $V_{\nu L} = U^\dagger$. The LEP LFU constraint in figure 1 would no longer apply, widening the parameter space shown in the figure. The Z' would then couple only to quarks, neutrinos and muons (i.e. not to $\tau\tau$). This tweaked model is similar to the ‘ $33\mu\mu$ ’ model of ref. [47], except that the Z' would contain additional couplings to μ_R as well as to μ_L .

There are discrepancies with SM predictions at a similar level to $R_{K^{(*)}}$ (when measured in numbers of sigma) in $B \rightarrow D^{(*)}\tau\nu$ decays [60–63], which we have not addressed in our model. However, to explain this charged current, a different mass scale is required to the one that explains $R_{K^{(*)}}$: the mass divided by the square root of the product of two of its couplings is required to be around 3.4 TeV [4] in order to fit the data, i.e. an order of magnitude lighter, or a more strongly coupled particle. We note some ambitious models explaining, to some extent, both the $B \rightarrow D^{(*)}\tau\nu$ data and $R_{K^{(*)}}$ [64, 65] based on gauged vector leptoquarks¹⁹ [67, 68]. These models are rather involved (for example, they contain both a Z' and a leptoquark). We have limited the scope of our much simpler model and we ignore the $B \rightarrow D^{(*)}\tau\nu$ data, being content for now to explain only the neutral current discrepancies with SM predictions. If the charged current B discrepancies stand the test of time, clearly the model should be extended in order to take them into account.

The Third Family Hypercharge Model explains the $R_{K^{(*)}}$ measurements by predicting a Z' with flavour dependent couplings. The third family of fermions (and the Higgs doublet) has a $U(1)_F$ charge given by the hypercharge, resulting in a hierarchically massive third family of fermions and small CKM mixing elements. The precise constraints upon the model do depend upon choices in the fermion mixing parameters. We have showed, in one simple example case, an existence proof where the model passes the relevant current experimental tests. The model as a whole is fairly concise, requiring no additional fields to the SM, save for the $U(1)_F$ gauge field and a complex SM singlet scalar to spontaneously break the symmetry, and is moreover anomaly free.

The Third Family Hypercharge Model (and other models of its ilk), raise the exciting possibility of *providing a direct experimental probe (through measurements of Z' couplings) of mechanisms pertinent to the ‘fermion masses and mixings’ problem.*

¹⁸Such a $U(1)_F$ charge assignment remains anomaly-free.

¹⁹We notice the appearance of a Pati-Salam vector leptoquark in ref. [66] in order to explain the discrepant B -measurements.

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A Fermion rotation to the mass basis

Here, we detail the rotation of SM fermion fields to the mass basis in order to fix our conventions. We write

$$\begin{aligned} \mathbf{u}'_{\mathbf{L}} &= \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix}, & \mathbf{d}'_{\mathbf{L}} &= \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}, & \mathbf{n}'_{\mathbf{L}} &= \begin{pmatrix} \nu_{eL}' \\ \nu_{\mu L}' \\ \nu_{\tau L}' \end{pmatrix}, & \mathbf{e}'_{\mathbf{L}} &= \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix}, \\ \mathbf{u}'_{\mathbf{R}} &= \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix}, & \mathbf{d}'_{\mathbf{R}} &= \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}, & \mathbf{e}'_{\mathbf{R}} &= \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}, \end{aligned}$$

along with the SM fermionic electroweak doublets

$$\mathbf{Q}'_{\mathbf{L}i} = \begin{pmatrix} \mathbf{u}'_{\mathbf{L}i} \\ \mathbf{d}'_{\mathbf{L}i} \end{pmatrix}, \quad \mathbf{L}'_{\mathbf{L}i} = \begin{pmatrix} \mathbf{n}'_{\mathbf{L}i} \\ \mathbf{e}'_{\mathbf{L}i} \end{pmatrix}.$$

The SM fermions acquire their mass through the terms

$$-\mathcal{L}_Y = \overline{\mathbf{Q}'_{\mathbf{L}}} Y_u H \mathbf{u}'_{\mathbf{R}} + \overline{\mathbf{Q}'_{\mathbf{L}}} Y_d H^c \mathbf{d}'_{\mathbf{R}} + \overline{\mathbf{L}'_{\mathbf{L}}} Y_e H^c \mathbf{e}'_{\mathbf{R}} + \frac{1}{2} \left(\overline{\mathbf{L}'_{\mathbf{L}}}^c H^\dagger \right) M^{-1} \left(\mathbf{L}'_{\mathbf{L}} H^\dagger \right) + \text{H.c.}, \quad (\text{A.1})$$

where Y_u , Y_d and Y_e are dimensionless complex coupling constants, each written as a 3 by 3 matrix in family space. These will have large 33 elements and smaller off-diagonal elements, in agreement with eq. (2.1). The matrix M^{-1} is a 3 by 3 matrix of mass dimension -1. After electroweak symmetry breaking, the terms in eq. (A.1) become the fermion mass terms plus some Higgs interactions:

$$\begin{aligned} -\mathcal{L}_Y &= \overline{\mathbf{u}'_{\mathbf{L}}} V_{uL} V_{uL}^\dagger m_u V_{uR} V_{uR}^\dagger \mathbf{u}'_{\mathbf{R}} + \overline{\mathbf{d}'_{\mathbf{L}}} V_{dL} V_{dL}^\dagger m_d V_{dR} V_{dR}^\dagger \mathbf{d}'_{\mathbf{R}} + \\ &\quad \overline{\mathbf{e}'_{\mathbf{L}}} V_{eL} V_{eL}^\dagger m_e V_{eR} V_{eR}^\dagger \mathbf{e}'_{\mathbf{R}} + \overline{\mathbf{n}'_{\mathbf{L}}}^c V_{\nu L}^* V_{\nu L}^T m_\nu V_{\nu L} V_{\nu L}^\dagger \mathbf{n}'_{\mathbf{L}} + \text{H.c.} + \dots \end{aligned} \quad (\text{A.2})$$

where V_{IL} and V_{IR} are 3 by 3 unitary matrices for each species I , $\mathbf{n}'_{\mathbf{L}}{}^c$ is the charge conjugate of the left-handed neutrino field, $m_u = vY_u$, $m_d = vY_d$, $m_e = vY_e$, and m_ν is the effective Majorana light neutrino mass matrix.

Choosing $V_{IL}^\dagger m_I V_{IR}$ to be diagonal, real and positive for the $I \in \{u, d, e\}$, and $V_{\nu L}^T m_\nu V_{\nu L}$ to be diagonal, real and positive for the neutrinos (all in increasing order of mass toward the bottom right of the matrix), we can identify the *non* primed mass eigenstates

$$\begin{aligned} \mathbf{u}_{\mathbf{R}} &\equiv V_{uR}^\dagger \mathbf{u}'_{\mathbf{R}}, & \mathbf{u}_{\mathbf{L}} &\equiv V_{uL}^\dagger \mathbf{u}'_{\mathbf{L}}, & \mathbf{d}_{\mathbf{R}} &\equiv V_{dR}^\dagger \mathbf{d}'_{\mathbf{R}}, & \mathbf{d}_{\mathbf{L}} &\equiv V_{dL}^\dagger \mathbf{d}'_{\mathbf{L}}, \\ \mathbf{e}_{\mathbf{R}} &\equiv V_{eR}^\dagger \mathbf{e}'_{\mathbf{R}}, & \mathbf{e}_{\mathbf{L}} &\equiv V_{eL}^\dagger \mathbf{e}'_{\mathbf{L}}, & \mathbf{n}_{\mathbf{L}} &\equiv V_{\nu L}^\dagger \mathbf{n}'_{\mathbf{L}}. \end{aligned} \quad (\text{A.3})$$

We may then identify the Cabibbo-Kobayashi-Maskawa matrix (CKM) V and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U :

$$V = V_{uL}^\dagger V_{dL}, \quad U = V_{\nu L}^\dagger V_{eL}. \quad (\text{A.4})$$

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References

- [1] LHCb collaboration, *Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **113** (2014) 151601 [[arXiv:1406.6482](#)] [[INSPIRE](#)].
- [2] LHCb collaboration, *Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays*, *JHEP* **08** (2017) 055 [[arXiv:1705.05802](#)] [[INSPIRE](#)].
- [3] G. Hiller and F. Krüger, *More model-independent analysis of $b \rightarrow s$ processes*, *Phys. Rev. D* **69** (2004) 074020 [[hep-ph/0310219](#)] [[INSPIRE](#)].
- [4] G. D’Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre et al., *Flavour anomalies after the R_{K^*} measurement*, *JHEP* **09** (2017) 010 [[arXiv:1704.05438](#)] [[INSPIRE](#)].
- [5] B. Gripaios, M. Nardecchia and S.A. Renner, *Linear flavour violation and anomalies in B physics*, *JHEP* **06** (2016) 083 [[arXiv:1509.05020](#)] [[INSPIRE](#)].
- [6] A. Carmona and F. Goertz, *Lepton Flavor and Nonuniversality from Minimal Composite Higgs Setups*, *Phys. Rev. Lett.* **116** (2016) 251801 [[arXiv:1510.07658](#)] [[INSPIRE](#)].
- [7] A. Carmona and F. Goertz, *Recent B physics anomalies: a first hint for compositeness?*, *Eur. Phys. J. C* **78** (2018) 979 [[arXiv:1712.02536](#)] [[INSPIRE](#)].
- [8] B. Gripaios, M. Nardecchia and S.A. Renner, *Composite leptiquarks and anomalies in B -meson decays*, *JHEP* **05** (2015) 006 [[arXiv:1412.1791](#)] [[INSPIRE](#)].
- [9] I. Garcia Garcia, *LHCb anomalies from a natural perspective*, *JHEP* **03** (2017) 040 [[arXiv:1611.03507](#)] [[INSPIRE](#)].
- [10] M. Carena, E. Megías, M. Quiros and C. Wagner, *$R_{D^{(*)}}$ in custodial warped space*, [[arXiv:1809.01107](#)] [[INSPIRE](#)].
- [11] R. Gauld, F. Goertz and U. Haisch, *On minimal Z' explanations of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly*, *Phys. Rev. D* **89** (2014) 015005 [[arXiv:1308.1959](#)] [[INSPIRE](#)].
- [12] A.J. Buras, F. De Fazio and J. Girrbach, *331 models facing new $b \rightarrow s \mu^+ \mu^-$ data*, *JHEP* **02** (2014) 112 [[arXiv:1311.6729](#)] [[INSPIRE](#)].
- [13] A.J. Buras and J. Girrbach, *Left-handed Z' and Z FCNC quark couplings facing new $b \rightarrow s \mu^+ \mu^-$ data*, *JHEP* **12** (2013) 009 [[arXiv:1309.2466](#)] [[INSPIRE](#)].
- [14] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Quark flavor transitions in $L_\mu - L_\tau$ models*, *Phys. Rev. D* **89** (2014) 095033 [[arXiv:1403.1269](#)] [[INSPIRE](#)].
- [15] A.J. Buras, F. De Fazio and J. Girrbach-Noe, *Z - Z' mixing and Z -mediated FCNCs in $SU(3)_C \times SU(3)_L \times U(1)_X$ models*, *JHEP* **08** (2014) 039 [[arXiv:1405.3850](#)] [[INSPIRE](#)].
- [16] A. Crivellin, G. D’Ambrosio and J. Heeck, *Explaining $h \rightarrow \mu^\pm \tau^\mp$, $B \rightarrow K^* \mu^+ \mu^-$ and $B \rightarrow K \mu^+ \mu^- / B \rightarrow K e^+ e^-$ in a two-Higgs-doublet model with gauged $L_\mu - L_\tau$* , *Phys. Rev. Lett.* **114** (2015) 151801 [[arXiv:1501.00993](#)] [[INSPIRE](#)].

- [17] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [[arXiv:1503.03477](#)] [[INSPIRE](#)].
- [18] D. Aristizabal Sierra, F. Staub and A. Vicente, *Shedding light on the $b \rightarrow s$ anomalies with a dark sector*, *Phys. Rev. D* **92** (2015) 015001 [[arXiv:1503.06077](#)] [[INSPIRE](#)].
- [19] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski and J. Rosiek, *Lepton-flavour violating B decays in generic Z' models*, *Phys. Rev. D* **92** (2015) 054013 [[arXiv:1504.07928](#)] [[INSPIRE](#)].
- [20] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, *Family nonuniversal Z' models with protected flavor-changing interactions*, *Phys. Rev. D* **92** (2015) 015007 [[arXiv:1505.03079](#)] [[INSPIRE](#)].
- [21] A. Greljo, G. Isidori and D. Marzocca, *On the breaking of Lepton Flavor Universality in B decays*, *JHEP* **07** (2015) 142 [[arXiv:1506.01705](#)] [[INSPIRE](#)].
- [22] W. Altmannshofer and I. Yavin, *Predictions for lepton flavor universality violation in rare B decays in models with gauged $L_\mu - L_\tau$* , *Phys. Rev. D* **92** (2015) 075022 [[arXiv:1508.07009](#)] [[INSPIRE](#)].
- [23] B. Allanach, F.S. Queiroz, A. Strumia and S. Sun, *Z' models for the $LHCb$ and $g - 2$ muon anomalies*, *Phys. Rev. D* **93** (2016) 055045 [Erratum *ibid.* **D 95** (2017) 119902] [[arXiv:1511.07447](#)] [[INSPIRE](#)].
- [24] A. Falkowski, M. Nardecchia and R. Ziegler, *Lepton Flavor Non-Universality in B -meson Decays from a $U(2)$ Flavor Model*, *JHEP* **11** (2015) 173 [[arXiv:1509.01249](#)] [[INSPIRE](#)].
- [25] C.-W. Chiang, X.-G. He and G. Valencia, *Z' model for $b \rightarrow s \ell \bar{\ell}$ flavor anomalies*, *Phys. Rev. D* **93** (2016) 074003 [[arXiv:1601.07328](#)] [[INSPIRE](#)].
- [26] D. Bečirević, O. Sumensari and R. Zukanovich Funchal, *Lepton flavor violation in exclusive $b \rightarrow s$ decays*, *Eur. Phys. J. C* **76** (2016) 134 [[arXiv:1602.00881](#)] [[INSPIRE](#)].
- [27] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Non-abelian gauge extensions for B -decay anomalies*, *Phys. Lett. B* **760** (2016) 214 [[arXiv:1604.03088](#)] [[INSPIRE](#)].
- [28] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Phenomenology of an $SU(2) \times SU(2) \times U(1)$ model with lepton-flavour non-universality*, *JHEP* **12** (2016) 059 [[arXiv:1608.01349](#)] [[INSPIRE](#)].
- [29] P. Ko, Y. Omura, Y. Shigekami and C. Yu, *$LHCb$ anomaly and B physics in flavored Z' models with flavored Higgs doublets*, *Phys. Rev. D* **95** (2017) 115040 [[arXiv:1702.08666](#)] [[INSPIRE](#)].
- [30] R. Alonso, P. Cox, C. Han and T.T. Yanagida, *Anomaly-free local horizontal symmetry and anomaly-full rare B -decays*, *Phys. Rev. D* **96** (2017) 071701 [[arXiv:1704.08158](#)] [[INSPIRE](#)].
- [31] R. Alonso, P. Cox, C. Han and T.T. Yanagida, *Flavoured $B - L$ local symmetry and anomalous rare B decays*, *Phys. Lett. B* **774** (2017) 643 [[arXiv:1705.03858](#)] [[INSPIRE](#)].
- [32] Y. Tang and Y.-L. Wu, *Flavor non-universal gauge interactions and anomalies in B -meson decays*, *Chin. Phys. C* **42** (2018) 033104 [[arXiv:1705.05643](#)] [[INSPIRE](#)].
- [33] C.-H. Chen and T. Nomura, *Penguin $b \rightarrow s \ell'^+ \ell'^-$ and B -meson anomalies in a gauged $L_\mu - L_\tau$* , *Phys. Lett. B* **777** (2018) 420 [[arXiv:1707.03249](#)] [[INSPIRE](#)].

- [34] G. Faisel and J. Tandean, *Connecting $b \rightarrow s\bar{\ell}\ell$ anomalies to enhanced rare nonleptonic \bar{B}_s^0 decays in Z' model*, *JHEP* **02** (2018) 074 [[arXiv:1710.11102](#)] [[INSPIRE](#)].
- [35] K. Fuyuto, H.-L. Li and J.-H. Yu, *Implications of hidden gauged $U(1)$ model for B anomalies*, *Phys. Rev. D* **97** (2018) 115003 [[arXiv:1712.06736](#)] [[INSPIRE](#)].
- [36] L. Bian, H.M. Lee and C.B. Park, *B -meson anomalies and Higgs physics in flavored $U(1)'$ model*, *Eur. Phys. J. C* **78** (2018) 306 [[arXiv:1711.08930](#)] [[INSPIRE](#)].
- [37] M. Abdullah, M. Dalchenko, B. Dutta, R. Eusebi, P. Huang, T. Kamon et al., *Bottom-quark fusion processes at the LHC for probing Z' models and B -meson decay anomalies*, *Phys. Rev. D* **97** (2018) 075035 [[arXiv:1707.07016](#)] [[INSPIRE](#)].
- [38] G.H. Duan, X. Fan, M. Frank, C. Han and J.M. Yang, *A minimal $U(1)'$ extension of MSSM in light of the B decay anomaly*, [arXiv:1808.04116](#) [[INSPIRE](#)].
- [39] C.-W. Chiang, J. Jiang, T. Li and Y.-R. Wang, *Top hypercharge*, *JHEP* **12** (2007) 001 [[arXiv:0710.1268](#)] [[INSPIRE](#)].
- [40] C.D. Froggatt and H.B. Nielsen, *Hierarchy of Quark Masses, Cabibbo Angles and CP-violation*, *Nucl. Phys. B* **147** (1979) 277 [[INSPIRE](#)].
- [41] J. Ellis, M. Fairbairn and P. Tunney, *Anomaly-Free Models for Flavour Anomalies*, *Eur. Phys. J. C* **78** (2018) 238 [[arXiv:1705.03447](#)] [[INSPIRE](#)].
- [42] C. Bonilla, T. Modak, R. Srivastava and J.W.F. Valle, *$U(1)_{B_3-3L_\mu}$ gauge symmetry as a simple description of $b \rightarrow s$ anomalies*, *Phys. Rev. D* **98** (2018) 095002 [[arXiv:1705.00915](#)] [[INSPIRE](#)].
- [43] D. Bhatia, S. Chakraborty and A. Dighe, *Neutrino mixing and R_K anomaly in $U(1)_X$ models: a bottom-up approach*, *JHEP* **03** (2017) 117 [[arXiv:1701.05825](#)] [[INSPIRE](#)].
- [44] A. Greljo and D. Marzocca, *High- p_T dilepton tails and flavor physics*, *Eur. Phys. J. C* **77** (2017) 548 [[arXiv:1704.09015](#)] [[INSPIRE](#)].
- [45] S.F. King, *$R_{K^{(*)}}$ and the origin of Yukawa couplings*, *JHEP* **09** (2018) 069 [[arXiv:1806.06780](#)] [[INSPIRE](#)].
- [46] F.F. Deppisch, S. Kulkarni, H. Päs and E. Schumacher, *Leptoquark patterns unifying neutrino masses, flavor anomalies and the diphoton excess*, *Phys. Rev. D* **94** (2016) 013003 [[arXiv:1603.07672](#)] [[INSPIRE](#)].
- [47] B.C. Allanach, B. Gripaios and T. You, *The case for future hadron colliders from $B \rightarrow K^{(*)}\mu^+\mu^-$ decays*, *JHEP* **03** (2018) 021 [[arXiv:1710.06363](#)] [[INSPIRE](#)].
- [48] T. Bandyopadhyay, G. Bhattacharyya, D. Das and A. Raychaudhuri, *Reappraisal of constraints on Z' models from unitarity and direct searches at the LHC*, *Phys. Rev. D* **98** (2018) 035027 [[arXiv:1803.07989](#)] [[INSPIRE](#)].
- [49] PARTICLE DATA GROUP collaboration, M. Tanabashi et al., *Review of Particle Physics*, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [50] X.-G. He and G. Valencia, *Ansatz for small FCNC with a non-universal Z -prime*, *Phys. Lett. B* **680** (2009) 72 [[arXiv:0907.4034](#)] [[INSPIRE](#)].
- [51] FERMILAB LATTICE, MILC collaboration, A. Bazavov et al., *$B_{(s)}^0$ -mixing matrix elements from lattice QCD for the Standard Model and beyond*, *Phys. Rev. D* **93** (2016) 113016 [[arXiv:1602.03560](#)] [[INSPIRE](#)].

- [52] L. Di Luzio, M. Kirk and A. Lenz, *Updated B_s -mixing constraints on new physics models for $b \rightarrow s\ell^+\ell^-$ anomalies*, *Phys. Rev. D* **97** (2018) 095035 [[arXiv:1712.06572](#)] [[INSPIRE](#)].
- [53] J. Charles, S. Descotes-Genon, Z. Ligeti, S. Monteil, M. Papucci and K. Trabelsi, *Future sensitivity to new physics in B_d, B_s and K mixings*, *Phys. Rev. D* **89** (2014) 033016 [[arXiv:1309.2293](#)] [[INSPIRE](#)].
- [54] CMS collaboration, *Search for associated production of a Z boson with a single top quark and for tZ flavour-changing interactions in pp collisions at $\sqrt{s} = 8$ TeV*, *JHEP* **07** (2017) 003 [[arXiv:1702.01404](#)] [[INSPIRE](#)].
- [55] CMS collaboration, *Search for high-mass resonances in dilepton final states in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *JHEP* **06** (2018) 120 [[arXiv:1803.06292](#)] [[INSPIRE](#)].
- [56] ATLAS collaboration, *Search for new high-mass phenomena in the dilepton final state using 36 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *JHEP* **10** (2017) 182 [[arXiv:1707.02424](#)] [[INSPIRE](#)].
- [57] S.L. Glashow, D. Guadagnoli and K. Lane, *Lepton Flavor Violation in B Decays?*, *Phys. Rev. Lett.* **114** (2015) 091801 [[arXiv:1411.0565](#)] [[INSPIRE](#)].
- [58] J. Albrecht, F. Bernlochner, M. Kenzie, S. Reichert, D. Straub and A. Tully, *Future prospects for exploring present day anomalies in flavour physics measurements with Belle II and LHCb*, [arXiv:1709.10308](#) [[INSPIRE](#)].
- [59] BELLE II collaboration, E. Kou et al., *The Belle II Physics Book*, [arXiv:1808.10567](#) [[INSPIRE](#)].
- [60] BABAR collaboration, J.P. Lees et al., *Measurement of an Excess of $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ Decays and Implications for Charged Higgs Bosons*, *Phys. Rev. D* **88** (2013) 072012 [[arXiv:1303.0571](#)] [[INSPIRE](#)].
- [61] LHCb collaboration, *Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\mu^-\bar{\nu}_\mu)$* , *Phys. Rev. Lett.* **115** (2015) 111803 [Erratum *ibid.* **115** (2015) 159901] [[arXiv:1506.08614](#)] [[INSPIRE](#)].
- [62] BELLE collaboration, S. Hirose et al., *Measurement of the τ lepton polarization and $R(D^*)$ in the decay $\bar{B} \rightarrow D^*\tau^-\bar{\nu}_\tau$* , *Phys. Rev. Lett.* **118** (2017) 211801 [[arXiv:1612.00529](#)] [[INSPIRE](#)].
- [63] LHCb collaboration, *Test of Lepton Flavor Universality by the measurement of the $B^0 \rightarrow D^{*-}\tau^+\nu_\tau$ branching fraction using three-prong τ decays*, *Phys. Rev. D* **97** (2018) 072013 [[arXiv:1711.02505](#)] [[INSPIRE](#)].
- [64] M. Bordone, C. Cornella, J. Fuentes-Martin and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, *Phys. Lett. B* **779** (2018) 317 [[arXiv:1712.01368](#)] [[INSPIRE](#)].
- [65] A. Greljo and B.A. Stefanek, *Third family quark-lepton unification at the TeV scale*, *Phys. Lett. B* **782** (2018) 131 [[arXiv:1802.04274](#)] [[INSPIRE](#)].
- [66] N. Assad, B. Fornal and B. Grinstein, *Baryon Number and Lepton Universality Violation in Leptoquark and Diquark Models*, *Phys. Lett. B* **777** (2018) 324 [[arXiv:1708.06350](#)] [[INSPIRE](#)].
- [67] L. Di Luzio, A. Greljo and M. Nardecchia, *Gauge leptoquark as the origin of B -physics anomalies*, *Phys. Rev. D* **96** (2017) 115011 [[arXiv:1708.08450](#)] [[INSPIRE](#)].
- [68] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B -physics anomalies: a guide to combined explanations*, *JHEP* **11** (2017) 044 [[arXiv:1706.07808](#)] [[INSPIRE](#)].